Calculus

Rules for Definite Integrals

1. Order of Integration: $\int_b^a f(x)dx = -\int_a^b f(x)dx$

*When you flip-flop the limits of an integral, the value of the integral switches sign.

2. Zero: $\int_a^a f(x)dx = 0$

*The integral of a function with no width is zero.

3. Constant Multiple Rule: $\int_a^b kf(x)dx = k\int_a^b f(x)dx$, for any number k

$\int_a^b -f(x)dx = -\int_a^b f(x)dx$, for $k = -1$

*The integral of a constant multiple of a function is the constant multiplied by the integral of the function.

4. Sum and Difference Rule: $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

*The integral of a sum or difference is the sum or difference of the integrals.

5. Additivity: $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

*The integral over two adjoining intervals is the integral of the entire interval.

6. Max-Min Inequality: If max f and min f are the maximum and minimum values of f on [a, b], then

$$\min f \cdot (b-a) \le \int_a^b f(x)dx \le \max f(b-a)$$

*The value of the integral is between the minimum and maximum rectangular areas for that interval.

7. Domination: $f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)dx \ge \int_a^b g(x)dx$

$f(x) \ge 0$ on $[a, b] \Rightarrow \int_a^b f(x)dx \ge 0$, $g = 0$

*A larger function has a larger integral value.

***Definition of Average (Mean) Value:**

If f is integrable on [a, b], then its **average (mean) value** on [a, b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)dx$.

***Mean Value Theorem for Integrals**:

If f is continuous on [a, b], then at some point c in [a, b], $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$.

***Antiderivative:** Remember that if $F'(x) = f(x)$, then $F(x)$ is an <u>antiderivative</u> of $f(x)$.

Thus, it follows that $\int_a^x f(t)dt = F(x) - F(a)$.

This essentially means that the derivative and integral "undo" each other.