

# Calculus

## Derivative Rules

### Name of Rule

### Rule

#### **Constant:**

\*The derivative of a constant is zero.

$$(c)' = 0$$

#### **Constant Multiple Rule:**

\*The derivative of a constant multiple of a function is the constant multiplied by the derivative of the function.

$$(cf)' = c \cdot f', (x)' = 1, \text{ and } (cx)' = c$$

#### **Sum and Difference Rule:**

\*The derivative of a sum or difference is the sum or difference of the derivatives.

$$(f \pm g)' = f' \pm g'$$

#### **Product Rule:**

\*The derivative of a product is the first function times the derivative of the second plus the second function times the derivative of the first.

$$(f \cdot g)' = f \cdot g' + g \cdot f'$$

#### **Multi-Product Rule:**

\*The derivative of a product is the first function times the derivative of the second plus the second function times the derivative of the first.

$$(f \cdot g \cdot h)' = f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$$

#### **Quotient Rule:**

\*The derivative of a quotient is the second function times the derivative of the first minus the first function times the derivative of the second, all over the second function squared.

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

#### **Power Rule:**

\*The derivative of a power is the power multiplied by the original function with one subtracted from the exponent.  
\*The **base** has the variable.

$$(f^n)' = n \cdot f^{n-1} \cdot f' \quad \text{Also, } (\sqrt{f})' = \frac{1}{2\sqrt{f}} \cdot f'$$

#### **Exponential:**

\*The **exponent** has the variable.

$$(a^f)' = \ln a \cdot a^f \cdot f'$$

$$\text{Also, } (e^x)' = e^x \text{ and } (e^f)' = e^f \cdot f'$$

#### **Doubly-Exponential:**

\*The base and the exponent both have variables. This combines the power and exponential rules.

$$(f^g)' = \underbrace{g \cdot f^{g-1} \cdot f'}_{\text{Power}} + \underbrace{\ln f \cdot f^g \cdot g'}_{\text{Exponential}}$$

**Logarithm:**

$$(\log_a |f|)' = \frac{1}{\ln a} \cdot \frac{1}{f} \cdot f'$$

**Natural Log:**

$$(\ln f)' = \frac{1}{f} \cdot f' \quad \text{Also, } (\ln x)' = \frac{1}{x}.$$

**Sine:**

$$(\sin f)' = \cos f \cdot f'$$

**Cosine:**

$$(\cos f)' = -\sin f \cdot f'$$

**Tangent:**

$$(\tan f)' = \sec^2 f \cdot f'$$

**Cotangent:**

$$(\cot f)' = -\csc^2 f \cdot f'$$

**Secant:**

$$(\sec f)' = \sec f \cdot \tan f \cdot f'$$

**Cosecant:**

$$(\csc f)' = -\csc f \cdot \cot f \cdot f'$$

**Arcsine:**

$$(\sin^{-1} f)' = \frac{1}{\sqrt{1-f^2}} \cdot f'$$

**Arccosine:**

$$(\cos^{-1} f)' = \frac{-1}{\sqrt{1-f^2}} \cdot f'$$

**Arctangent:**

$$(\tan^{-1} f)' = \frac{1}{1+f^2} \cdot f'$$

**Arccotangent:**

$$(\cot^{-1} f)' = \frac{-1}{1+f^2} \cdot f'$$

**Arcsecant:**

$$(\sec^{-1} f)' = \frac{1}{|f|} \cdot \frac{1}{\sqrt{f^2-1}} \cdot f'$$

\*Mathematicians cannot agree on this.

**Arccosecant:**

$$(\csc^{-1} f)' = \frac{-1}{|f|} \cdot \frac{1}{\sqrt{f^2-1}} \cdot f'$$

\*Mathematicians cannot agree on this.

**Absolute Value Rule:**

$$(|f|)' = \frac{f}{|f|} \cdot f'$$

\*This rule is rarely used.

Different Symbolism for “derivative”: instantaneous slope or rate of change,  $(\cos x)'$ ,  $\frac{d(\cos x)}{dx}$ ,  $y'$ ,  $f'(x)$ ,  $D_x$